MATTEO ROSATI - ROMA TRE UNIVERSITY, ITALY

Learning Quantum Channels without Input Control

Based on M. Fanizza, Y. Quek and M. R.

(arXiv: coming soon!)

Outline

- Background: learning classical and quantum systems
- Our problem: non-identical training samples
- Methods: one-time threshold search (for non-identical states)
- Applications: imaging, metrology and compiling

Statistical learning with classical and quantum systems

Classical Statistical Learning Theory

- Vapnik: $f_? : \mathcal{X} \to \mathcal{Y}$ $\quad \mathcal{C} = \{g : \mathcal{X} \to \mathcal{Y}\}$

V. N. Vapnik, IEEE Trans. Neural Networks, vol. 10, no. 5, pp. 988-999, 1999.

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in practice $\hat{R}(g) = \frac{1}{n} \sum_{i=1}^{n} L(f_?(x_i), g(x_i))$

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  D → VC dimension

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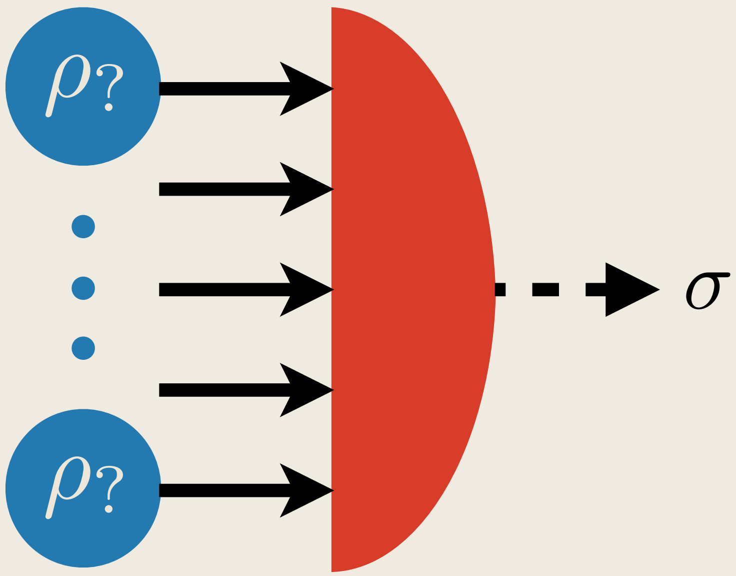

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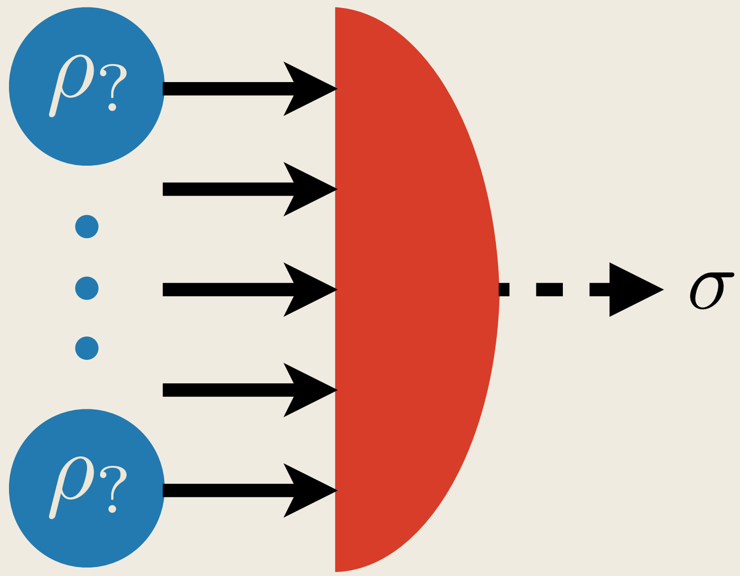

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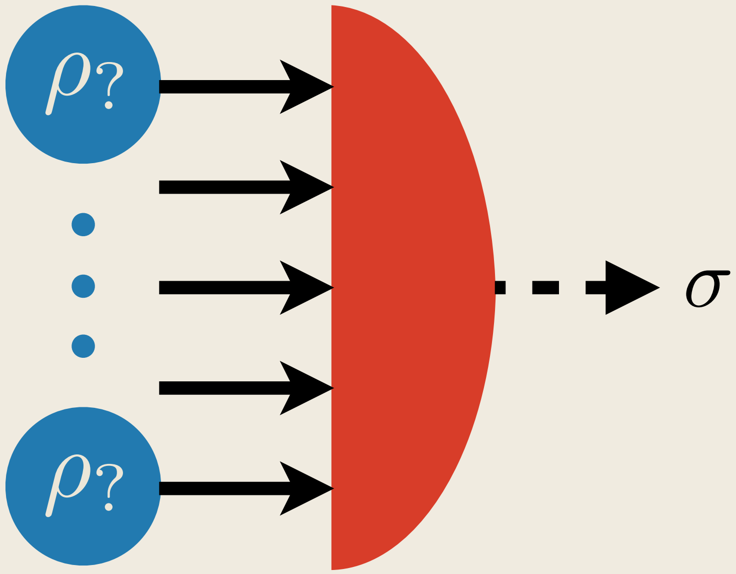

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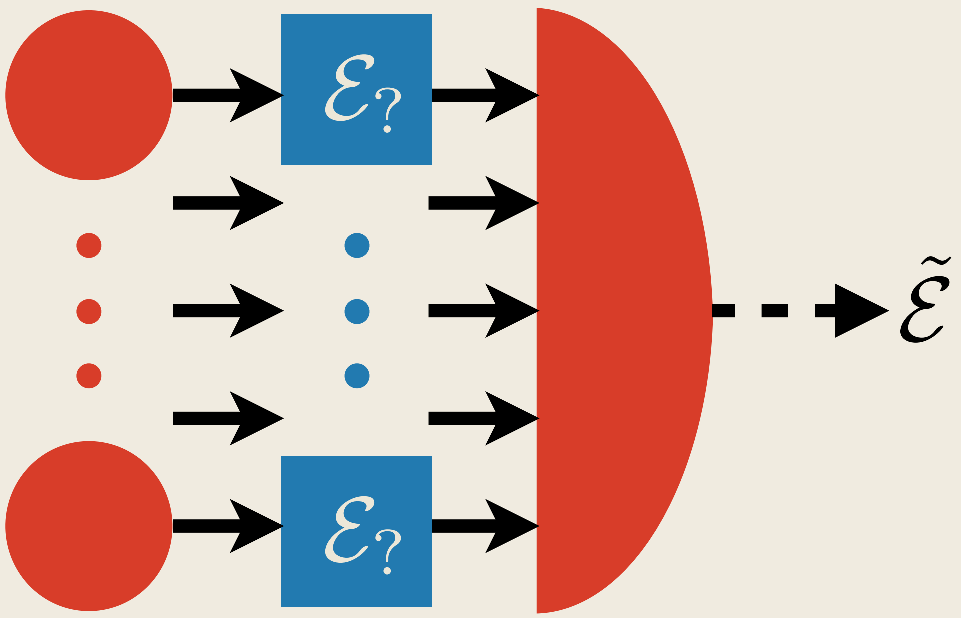

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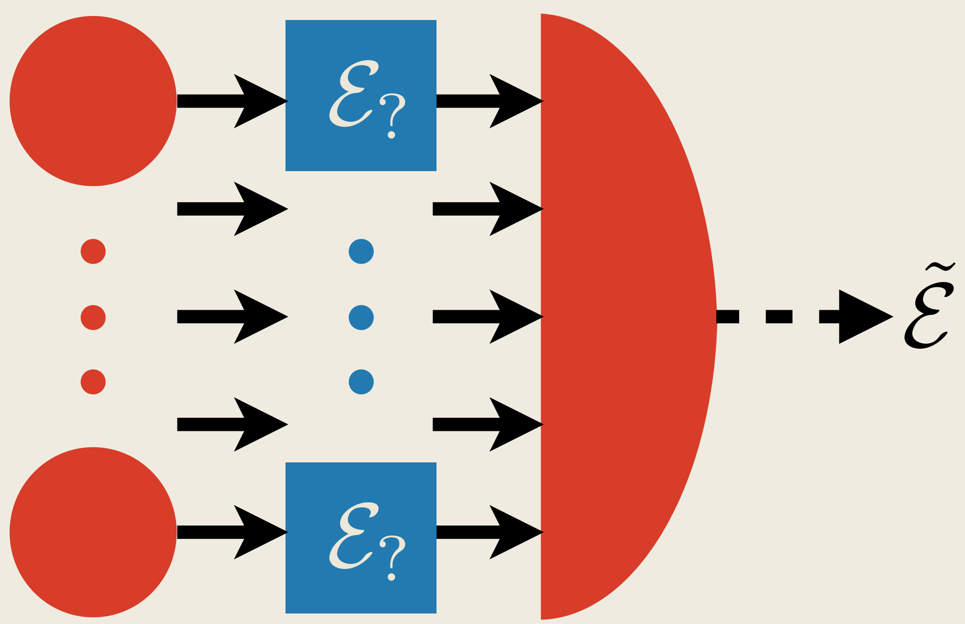

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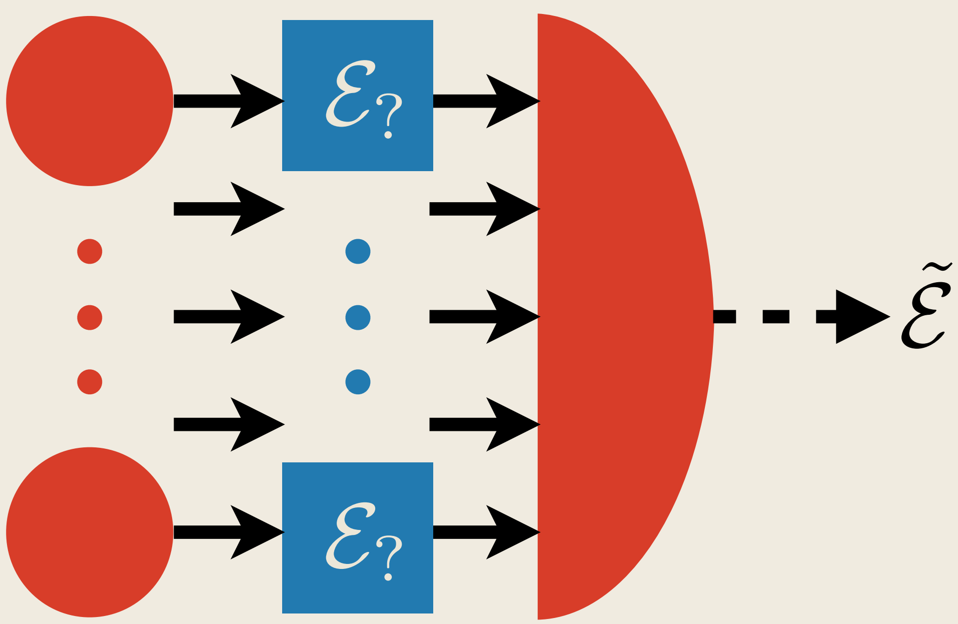

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Learning classical-quantum channels

Towards full Quantization

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M. Fanizza, Y. Quek and M. Rosati, "Learning quantum processes without input control", coming soon!

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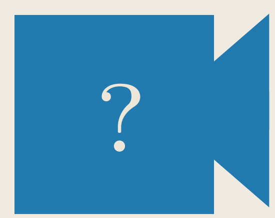

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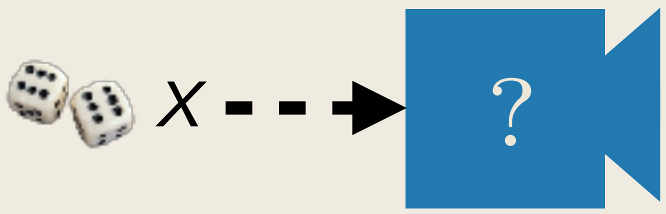

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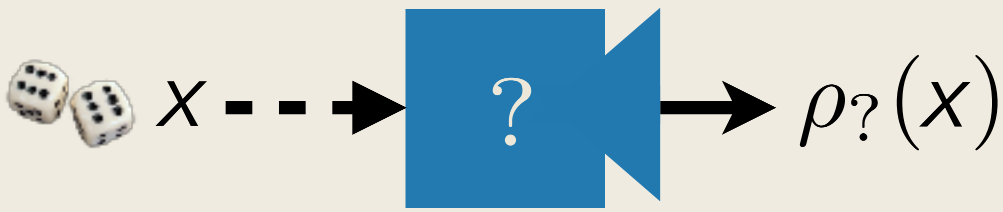

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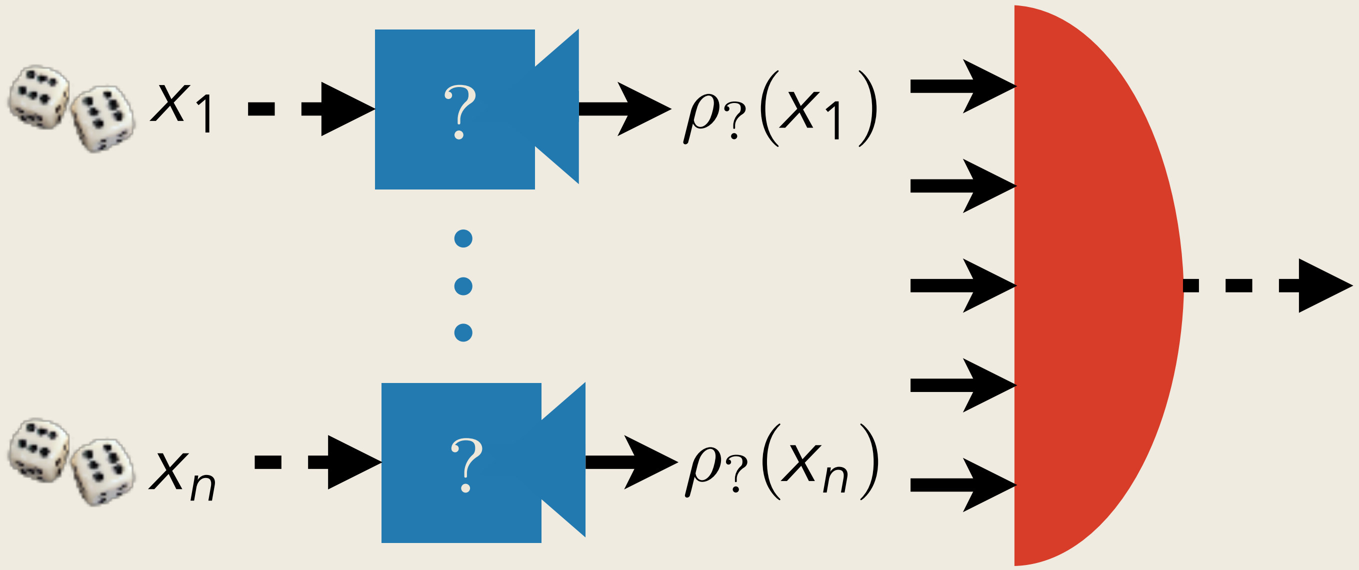

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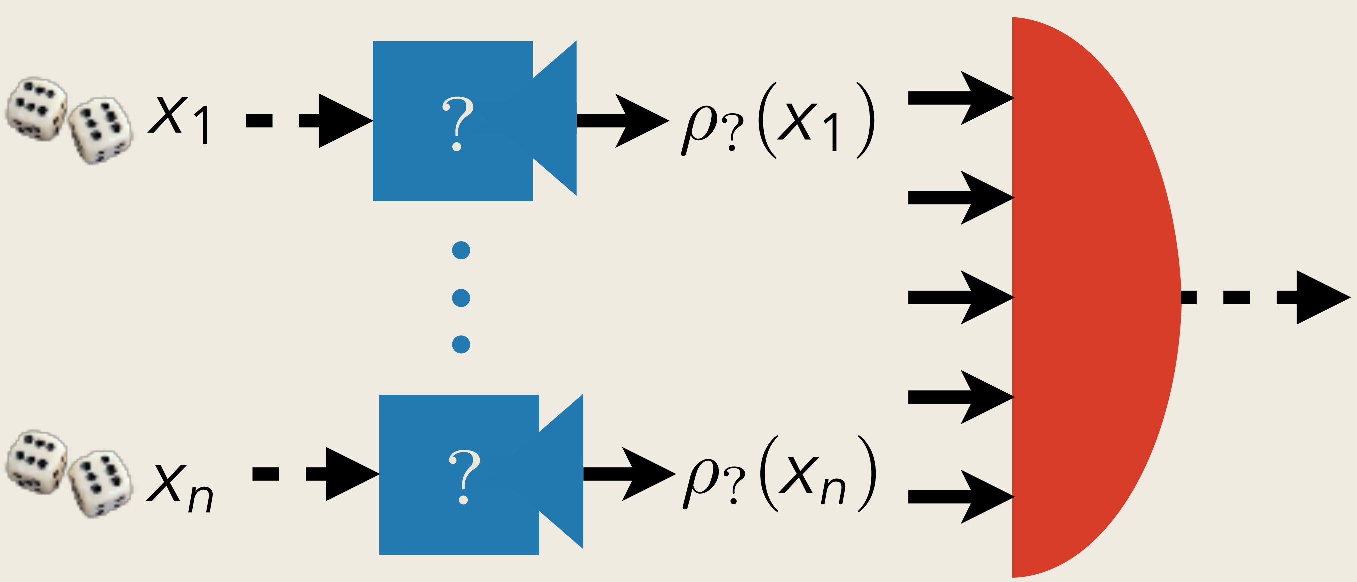

Task: find a good approximation of the classical-quantum process

$$x \mapsto \rho_?(x)$$

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$$\vec{x} \to \tilde{\mathcal{C}}|_{\vec{x}} \qquad \forall O \in \mathcal{C} \quad \exists \tilde{O} \in \tilde{\mathcal{C}}|_{\vec{x}} : \frac{1}{n} \sum_{i=1}^{n} ||O(x_i) - \tilde{O}(x_i)|| \leq \epsilon$$

Methods

Proof idea (projectors/pure states)

V. N. Vapnik and A. Y. Chervonenkis, 1971; S. Aaronson, 2018; Badescu and O'Donnell, 2020.
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ONE-TIME QUANTUM THRESHOLD SEARCH

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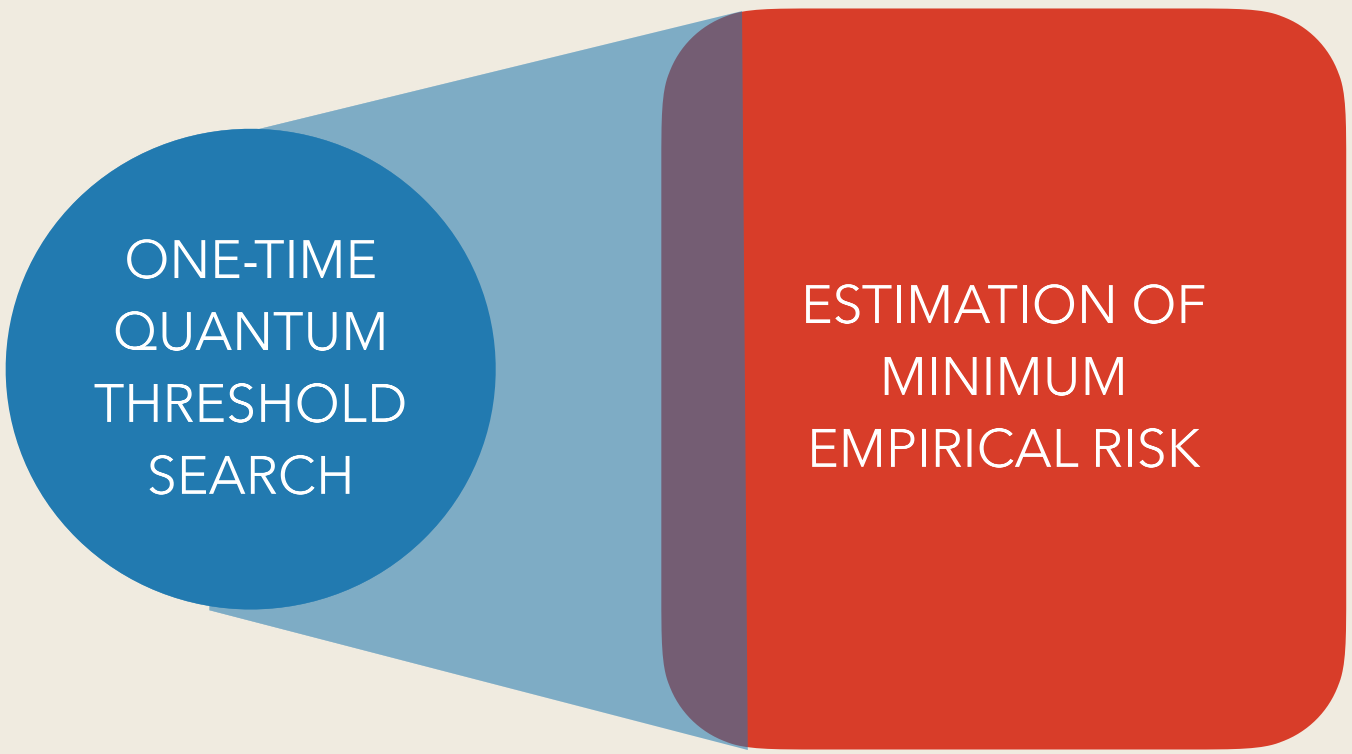

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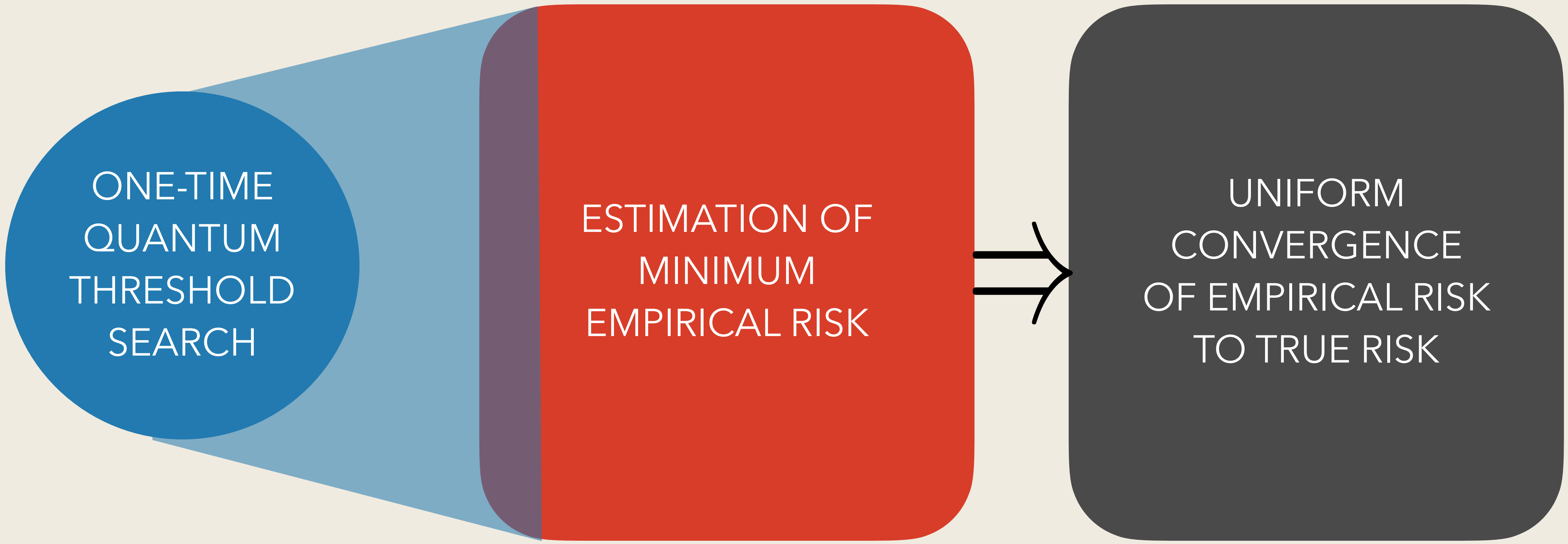

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$$P_{\vec{x}\in\mathcal{X}^n}(\exists \Pi \in \mathcal{C} : |\hat{R}_{\vec{x}}(\Pi) - R(\Pi)| \geq \epsilon) \leq 4\gamma_{1,\infty}(2n, \frac{\epsilon}{8}, \mathcal{C})e^{-\frac{n\epsilon^2}{32}}$$

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- Estimate minimum empirical risk

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- Key method:

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using $n = O(\mathrm{poly}(\epsilon^{-1}, \log \gamma))$

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- if "yes" then break
- if "no" then reuse the state for T_{j+1}

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Solution 3: iterate with binary search and "safety tests"

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Applications

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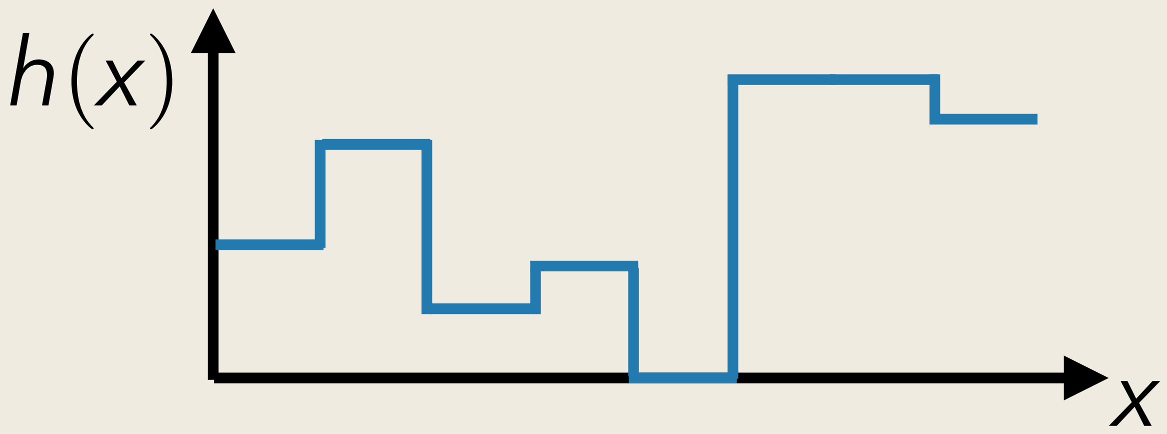

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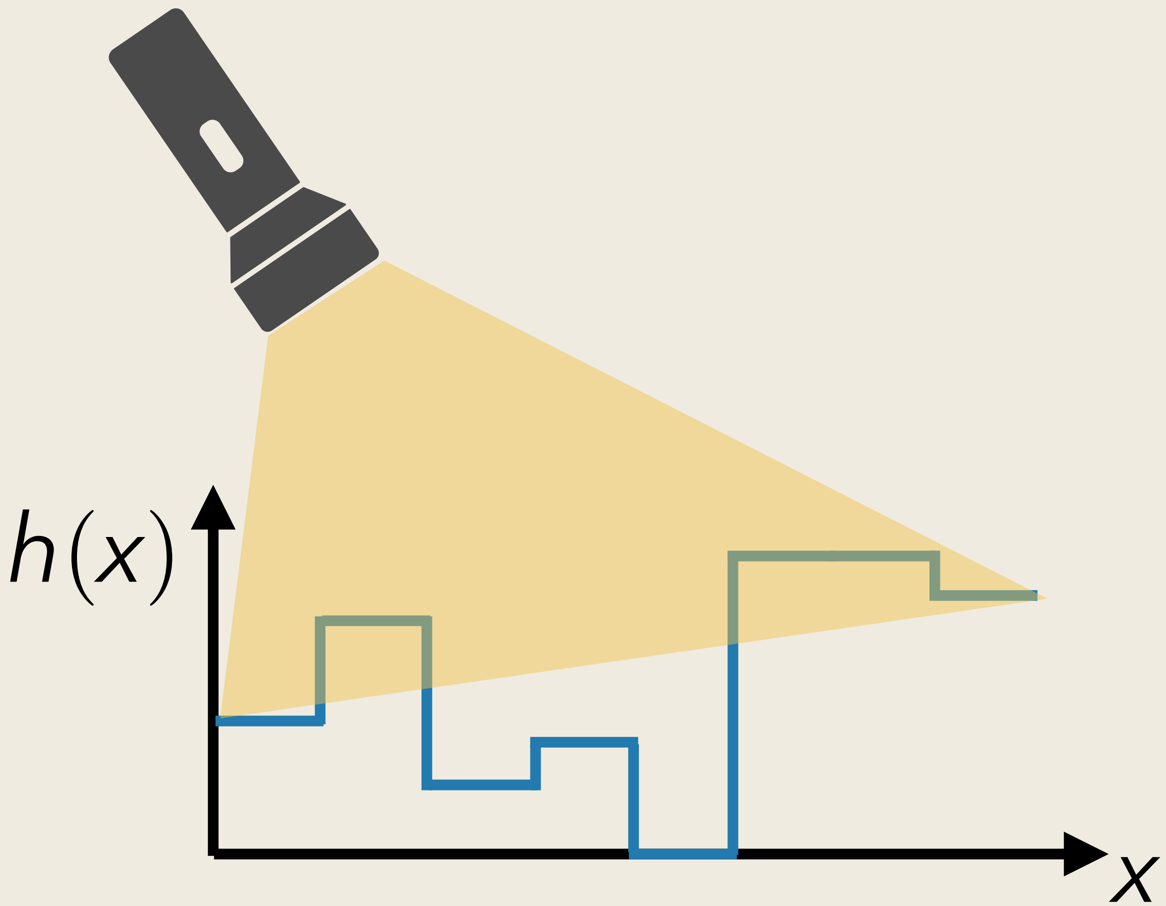

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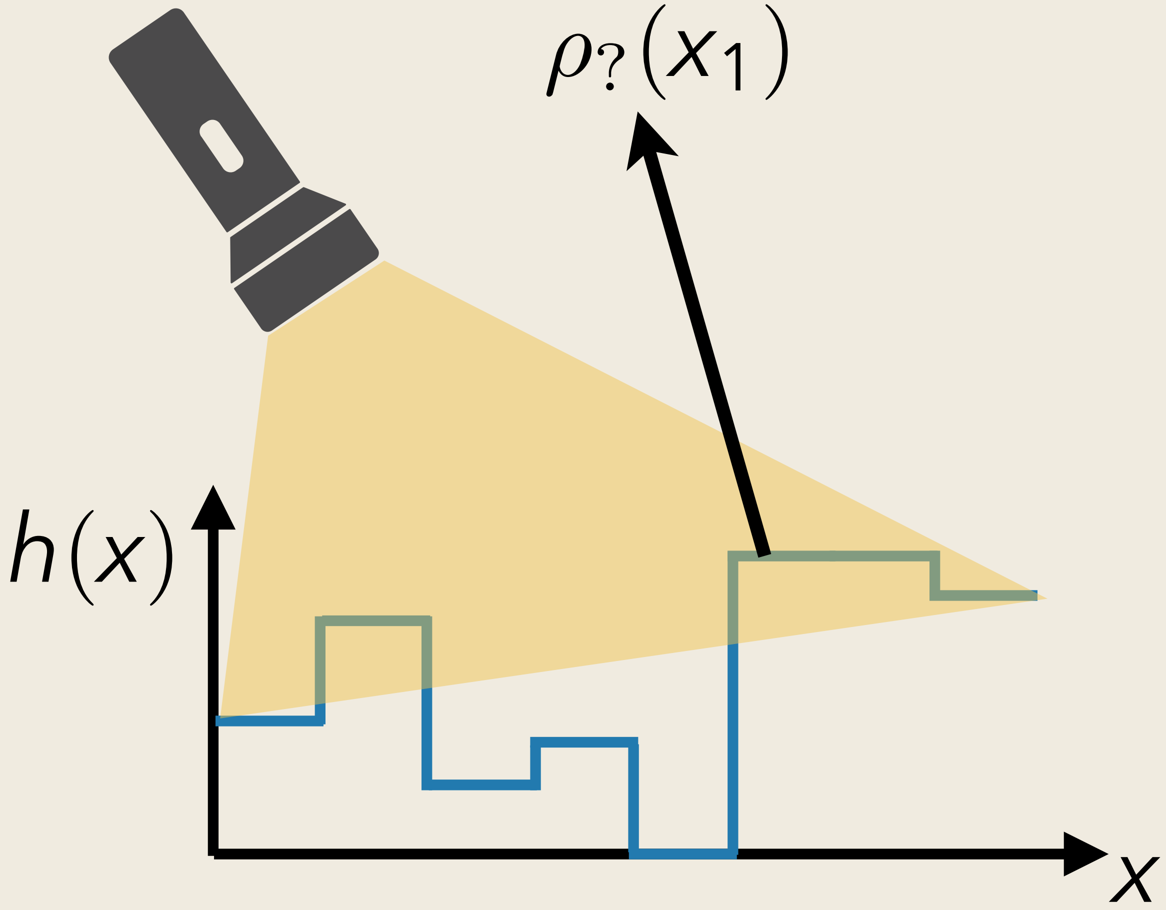

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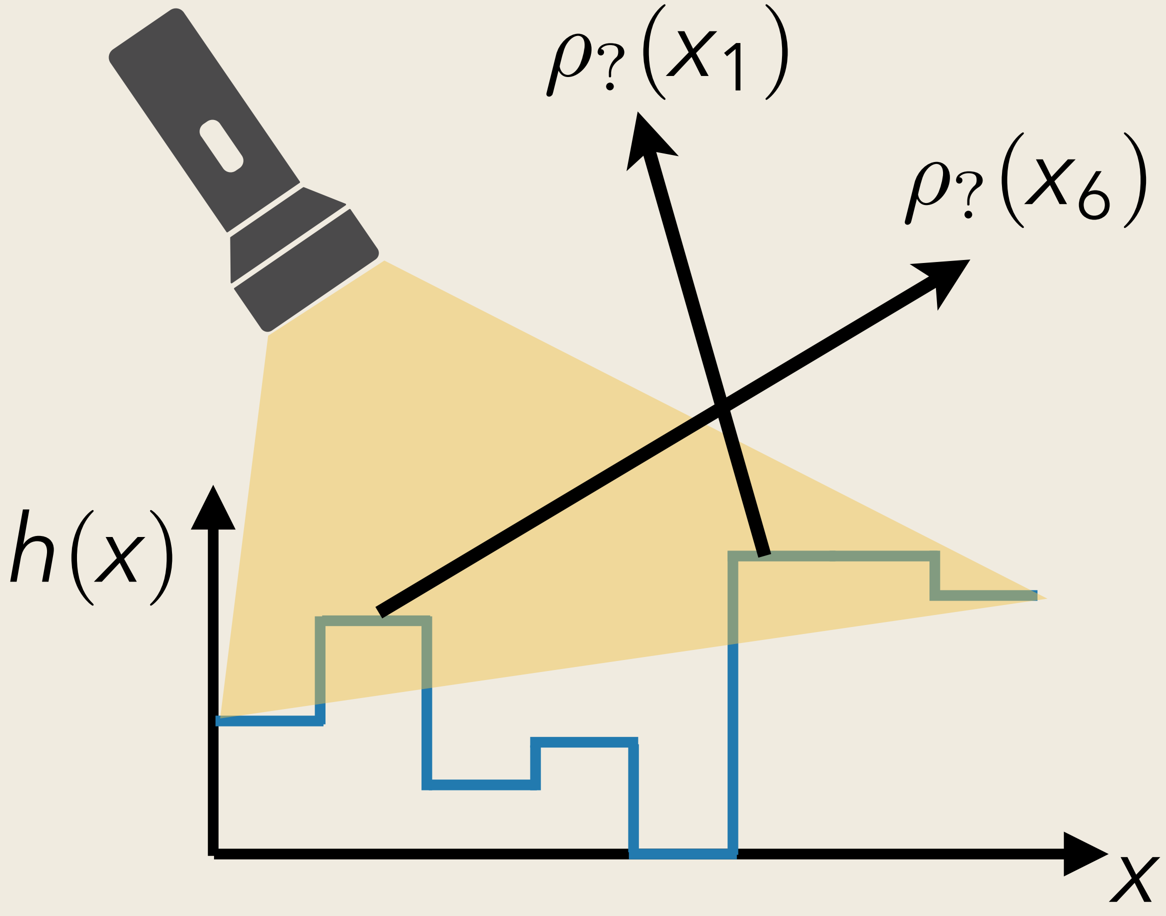

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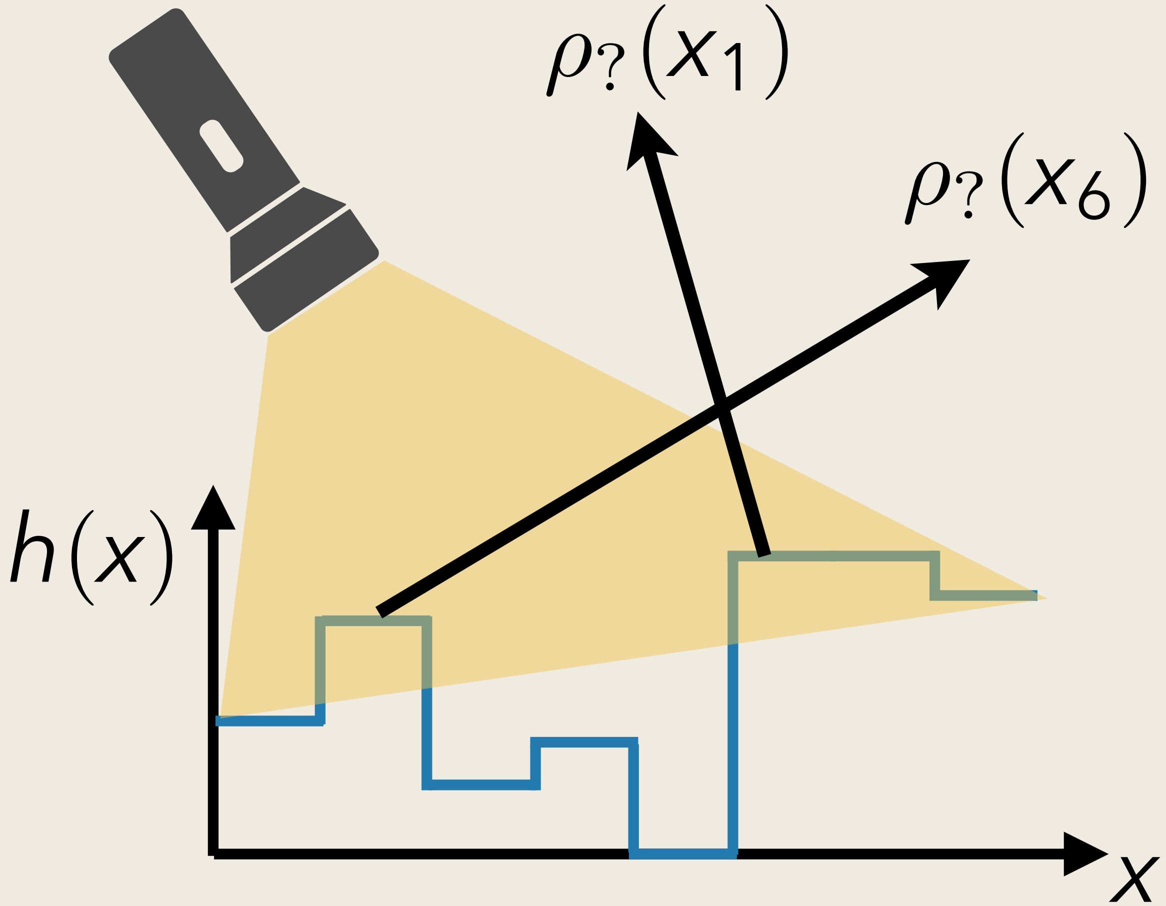

Learn the height profile as a function of the position

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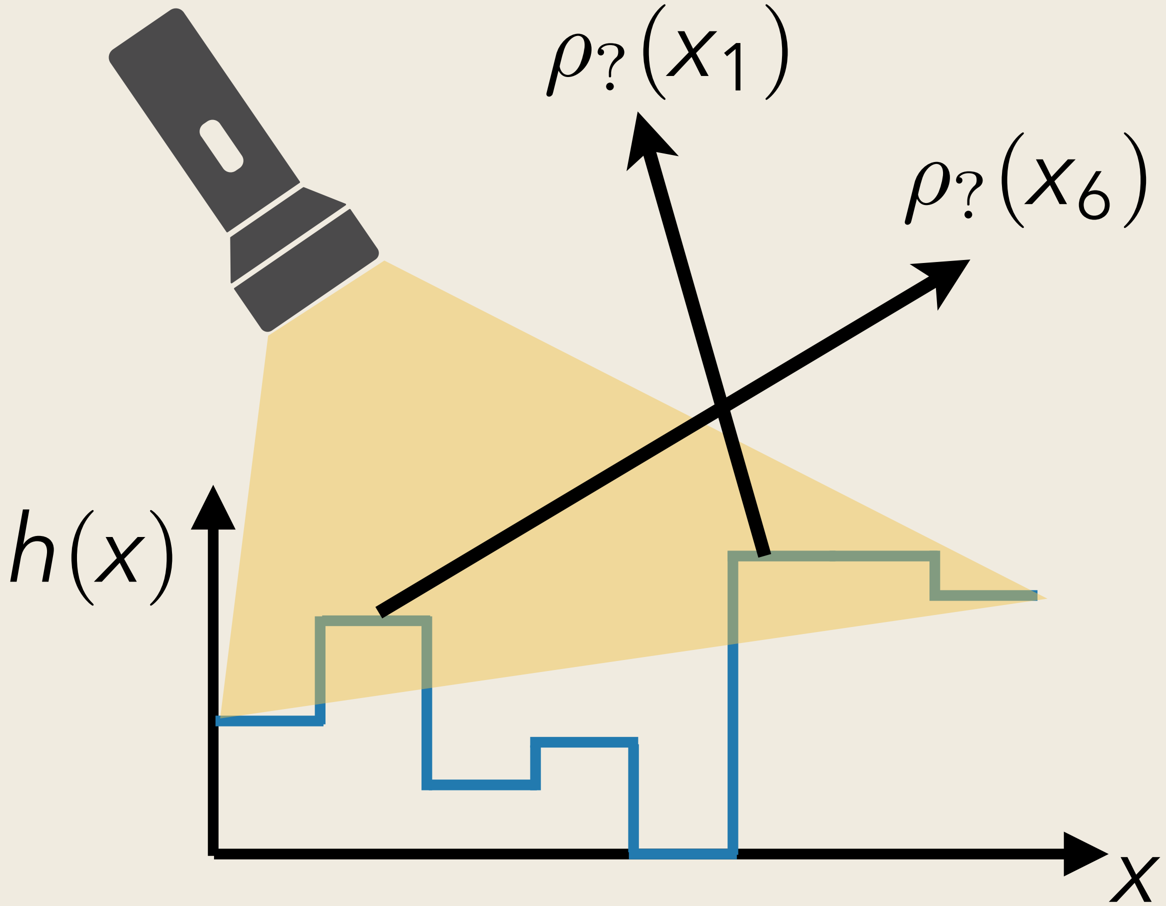

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$$\mathcal{C} = \{U^{h(x)}\psi(x)U^{\dagger h(x)}\}$$

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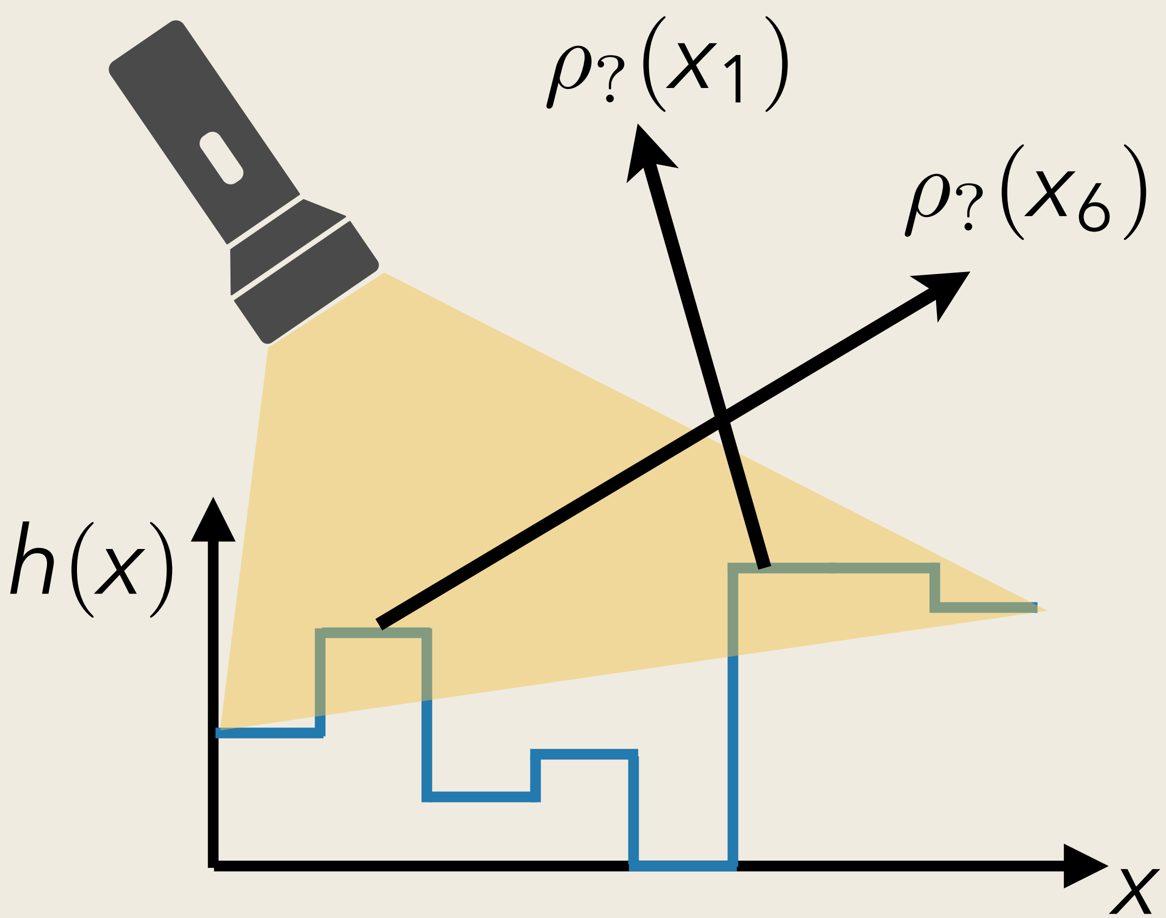

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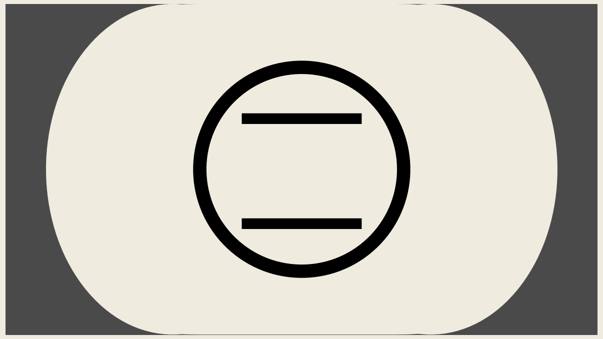

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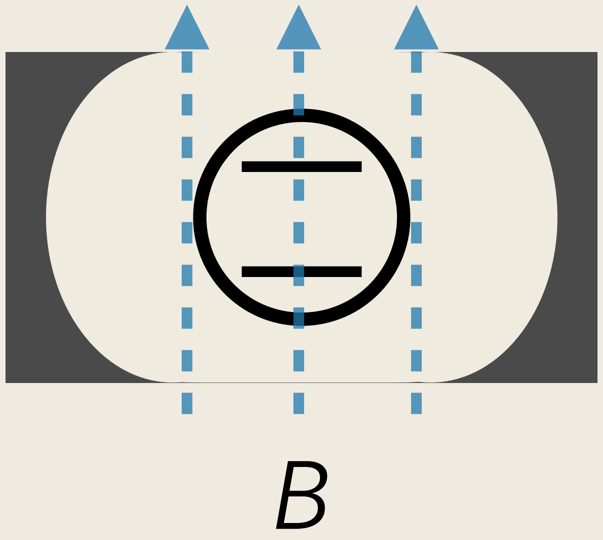

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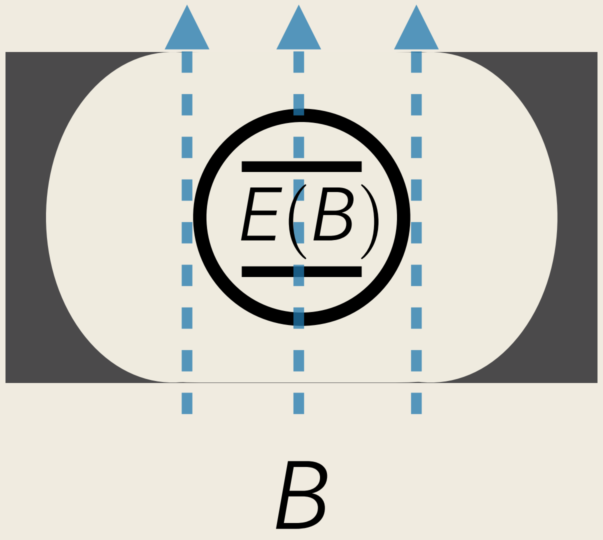

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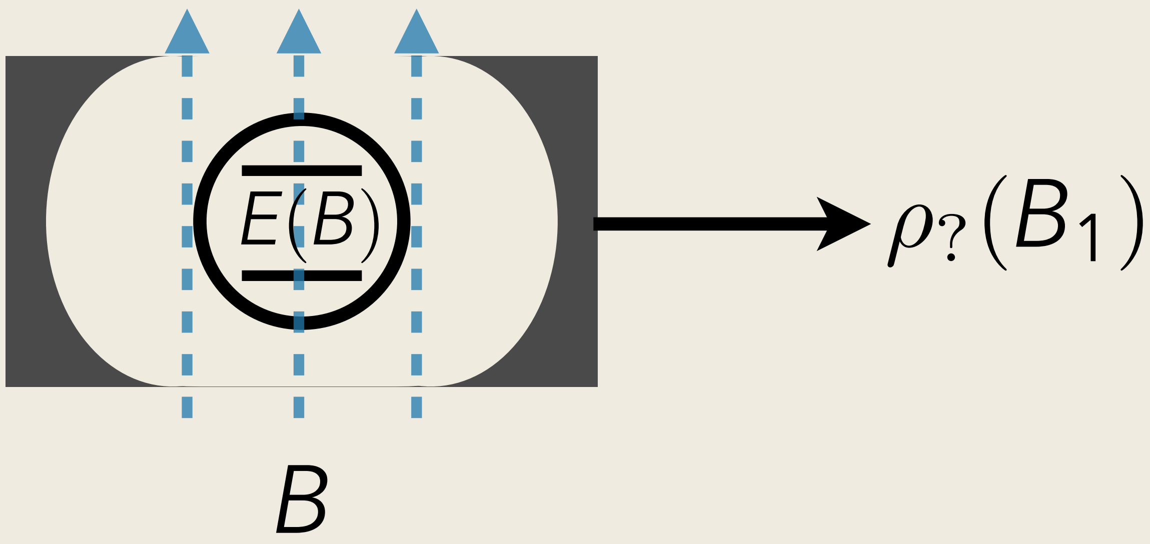

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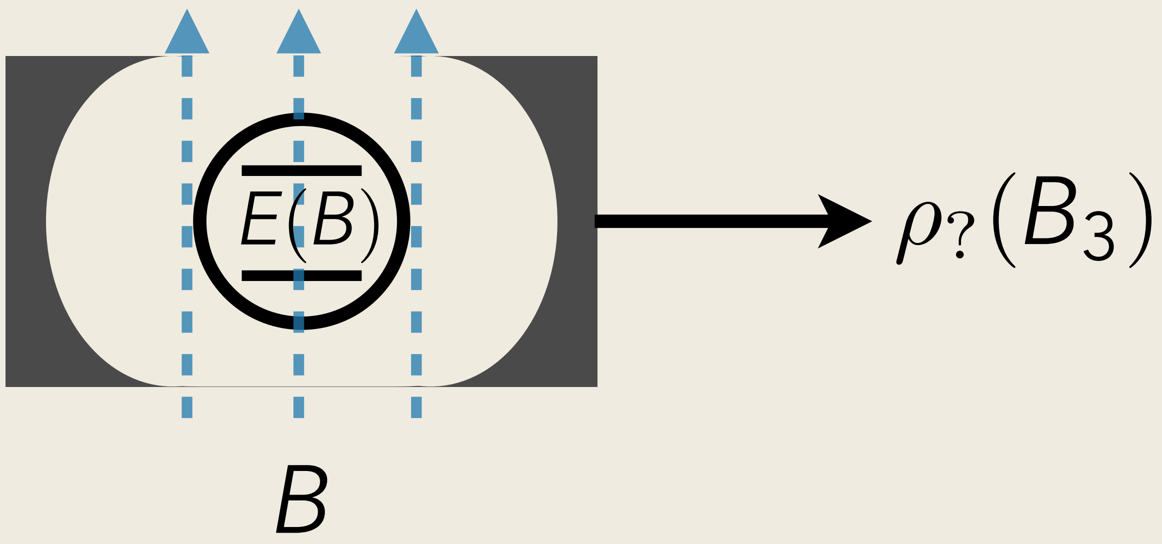

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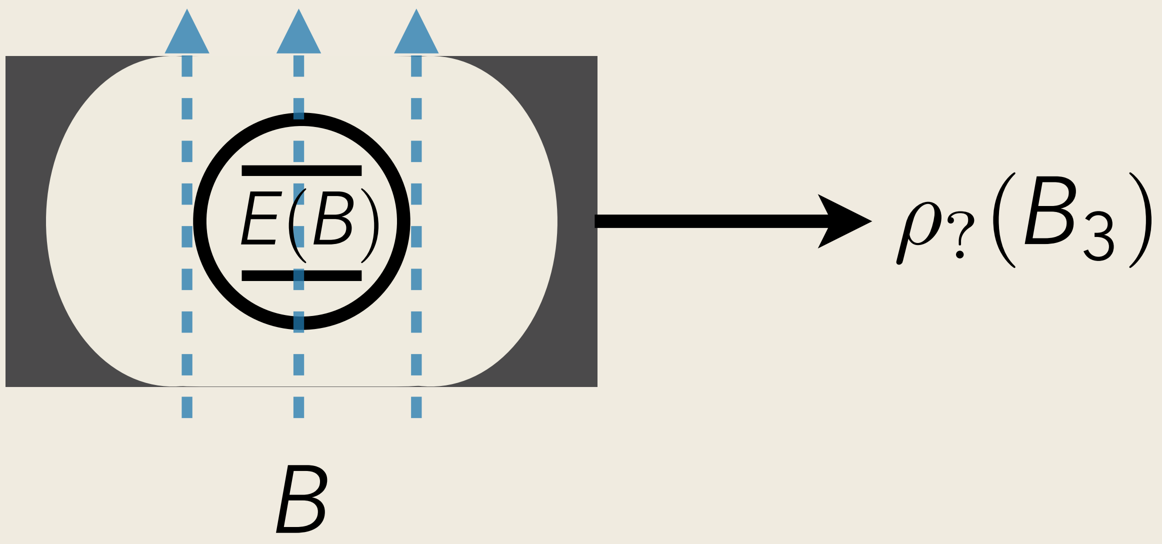

Learn the excited state energy as a function of the magnetic field

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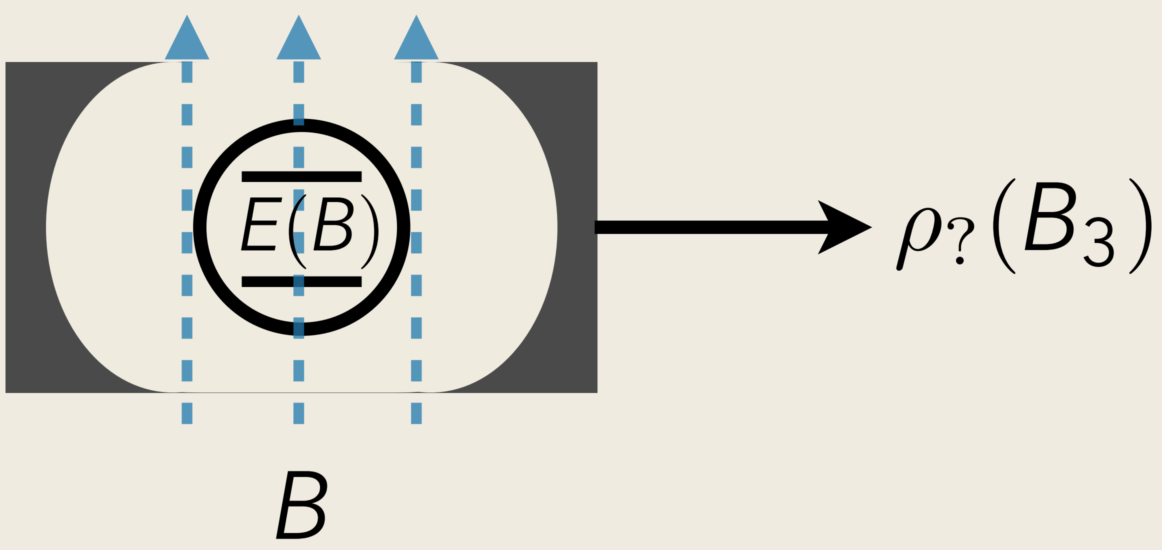

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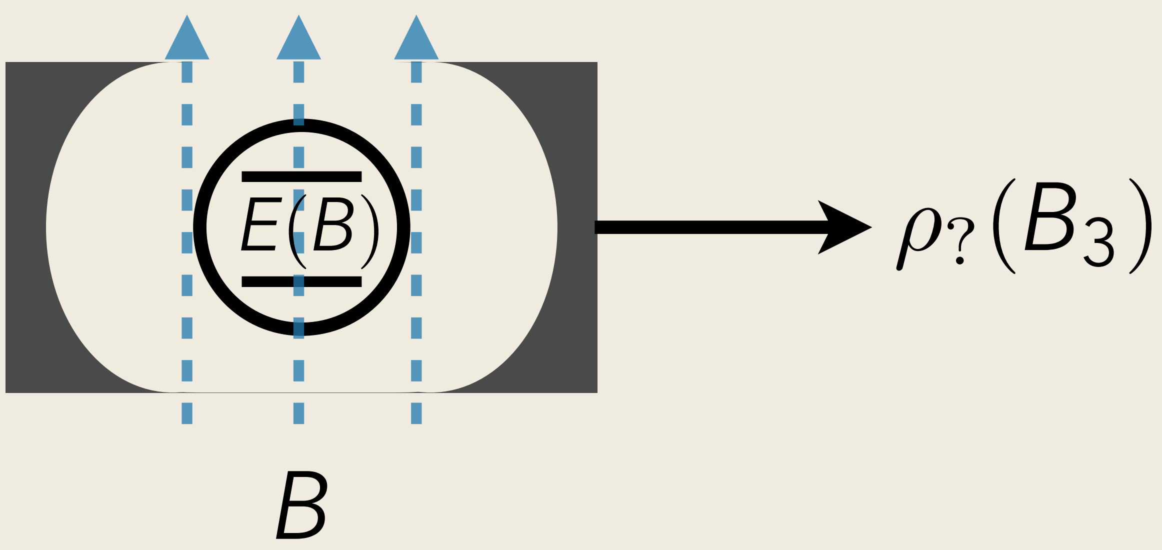

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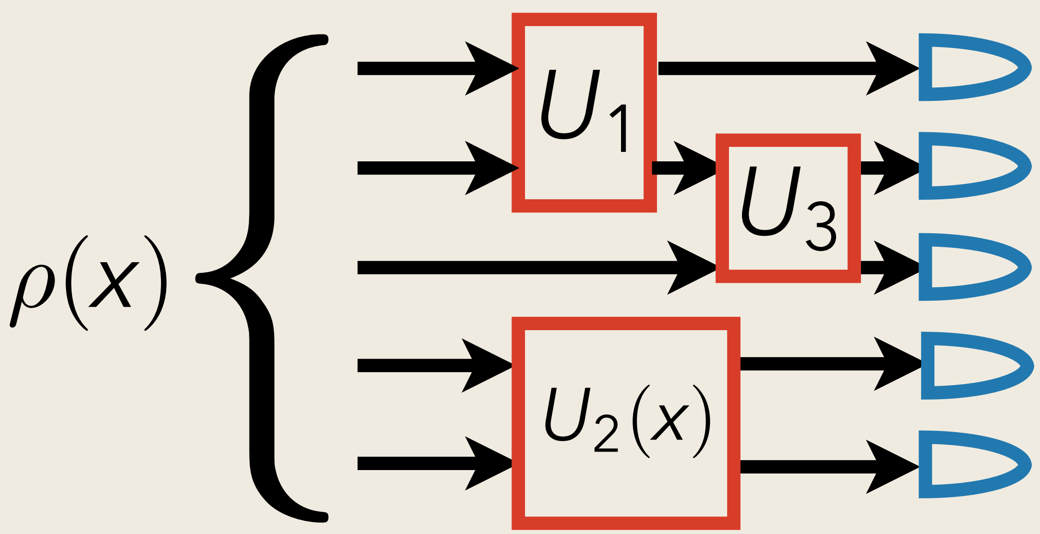

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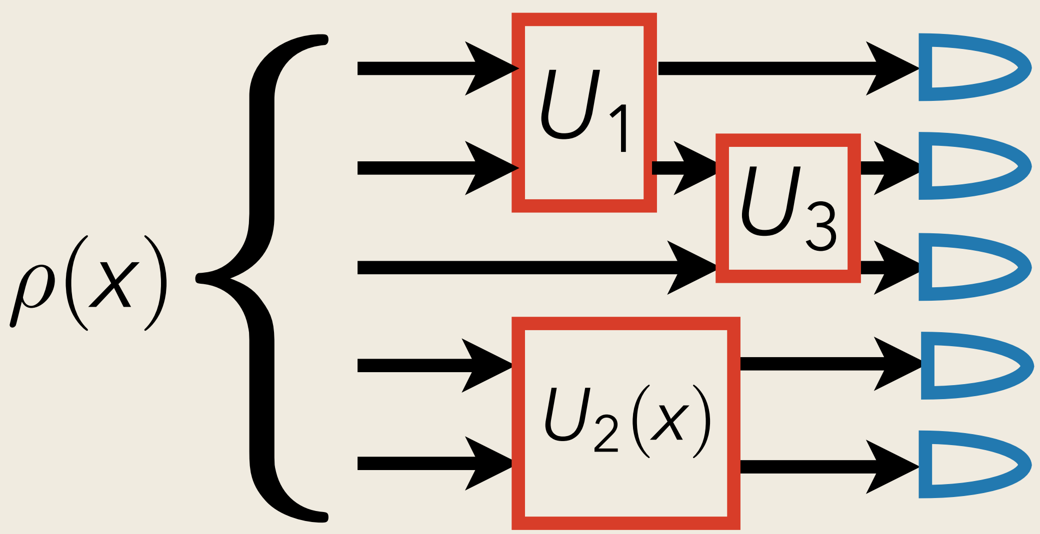

Learn the circuit as a function of the classical encoding parameters

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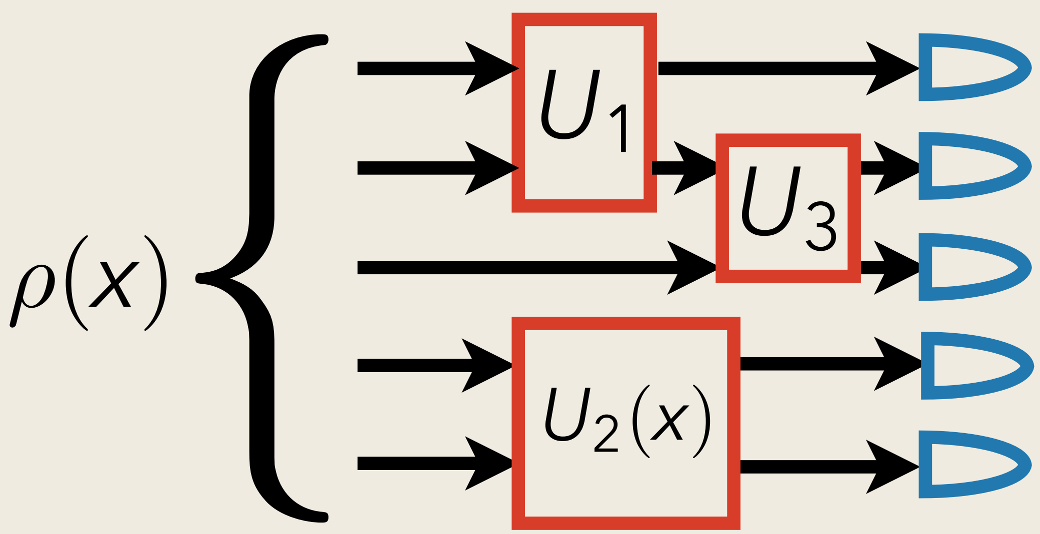

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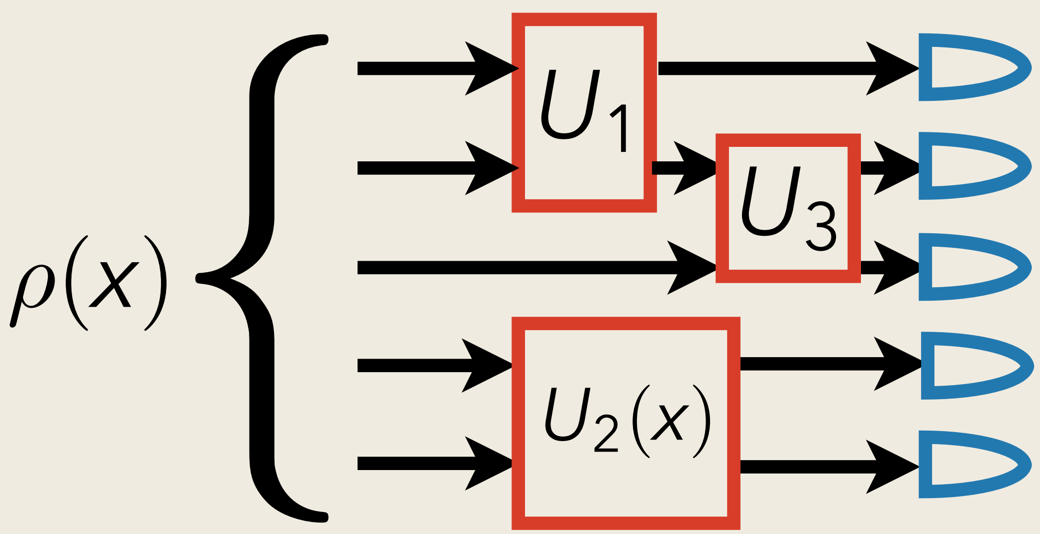

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Conclusions

- A general learnability condition for processes with classical input and quantum output
- Generalization of quantum threshold search to non-identical sources
- A learning framework for noisy or "random" experiments
- Applications: quantum imaging, hamiltonian learning…

see also: M. C. Caro, *Quantum Mach. Intell.*, vol. 3, no. 1, p. 18, Jun. 2021; K. M. Chung and H. H. Lin, *LIPIcs*, 2021, vol. 197.

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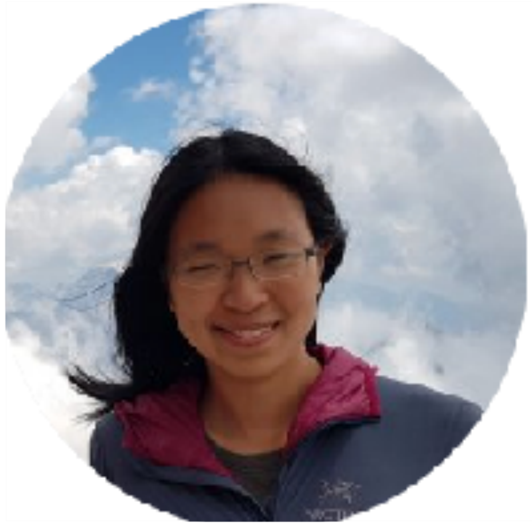
Y. Quek



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Funding

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