Detecting Quantum Capacities of Continuous-Variable Quantum Channels

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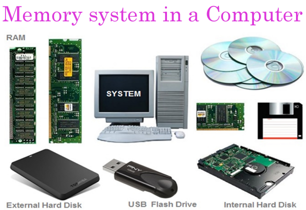
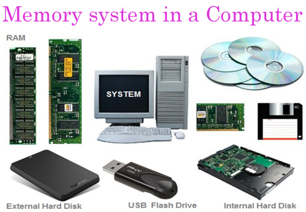
Quantum Communication and Quantum Memory



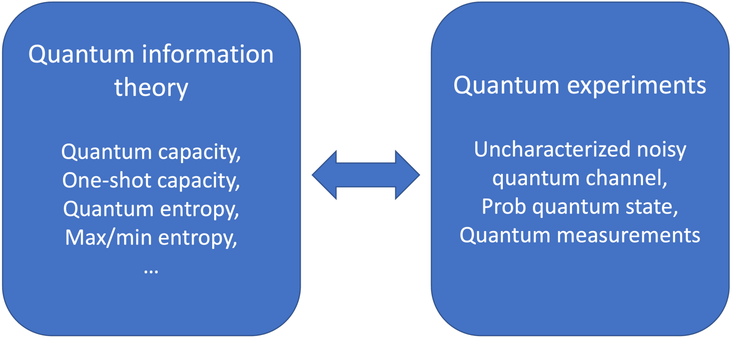
The central figure of merit for these devices is their quantum capacity, which quantifies how many qubits can be transmitted or stored.

Quantum Communication and Quantum Memory



The central figure of merit for these devices is their quantum capacity, which quantifies how many qubits can be transmitted or stored.
To assess the performance, one needs methods to estimate the quantum capacity from experimental data.

Quantum Information Theory vs Quantum Experiments



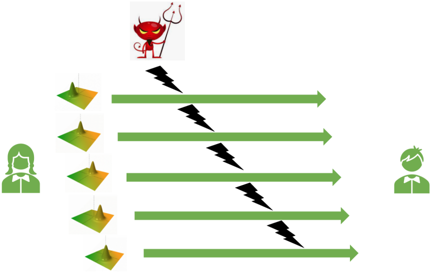
Similar to quantum process tomography, but we only care about detecting quantum capacity.

Difficulties in Detecting Quantum Capacities

- Explicit expressions for quantum capacity are only known for particularly simple noise models
- Calculation of the quantum capacity requires a classical description of the devices under consideration, which generally requires quantum process tomography
  For qubits: the sampling overhead scales exponentially with system size

Difficulties in Detecting Quantum Capacities

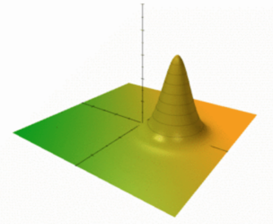
- Quantum noise in each use can fluctuate
- The devices can exhibit correlations across multiple uses, and can be under the control of an adversary.



Continuous-Variable Quantum Information

CV quantum information is carried by quantized electromagnetic field (quantum harmonic oscillator).

Differences: infinite-dimensional Hilbert space $\mathcal{H} := \mathrm{Span}\{|n\rangle\}_{n=0}^{\infty}$, observables have continuous spectra



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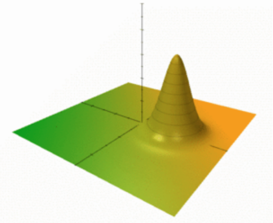
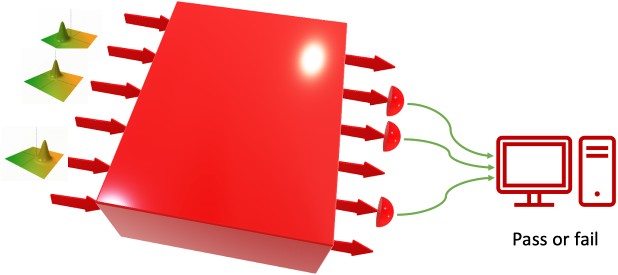


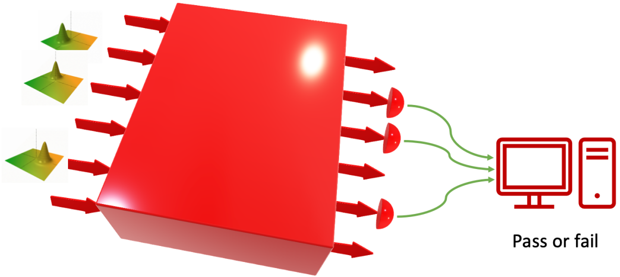
CV quantum systems are a promising platform for the realization of quantum technologies (using CV to encode qubits)

What we have done

We develop two protocols for estimating lower bounds on the quantum capacities of CV quantum channels, in the realistic scenario where the devices are used a finite number of times. One is for general non-iid noises and the other is for particular iid noises.



What we have done



Pass: get a lower bound on quantum capacity with high confidence, for example: average quantum capacity per mode is certified to be no less than 1.3 with a confidence 99.9%.

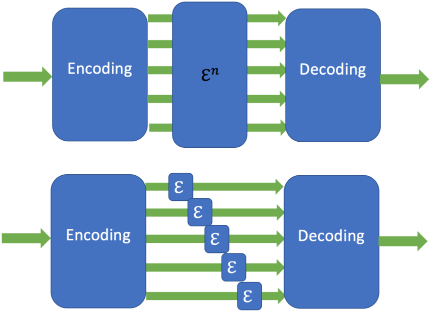
Failure: draw no conclusion on quantum capacity, cannot trust the channel

One-shot Quantum Capacity

One-shot quantum capacity is $Q^{\epsilon}(\mathcal{E}) := \max\{\log b | F(\mathcal{E}, b) \geq 1-\epsilon\}$, where b is $\dim(\bar{\mathcal{H}})$, $\bar{\mathcal{H}}$ is the encoded subspace, and

$$F(\mathcal{E}, b) := \max_{\bar{\mathcal{H}} \subset \mathcal{H}, \dim(\bar{\mathcal{H}})=b} \max_{\mathcal{D}} \min_{|\phi\rangle \in \bar{\mathcal{H}}} \langle\phi| \mathcal{D} \circ \mathcal{E}(|\phi\rangle\langle\phi|) |\phi\rangle .$$

When $\mathcal{E}^n = \mathcal{E}^{\otimes n}$, asymptotic quantum capacity is $Q(\mathcal{E}) = \lim_{\epsilon \to 0} \lim_{n \to \infty} Q^{\epsilon}(\mathcal{E}^{\otimes n})/n$

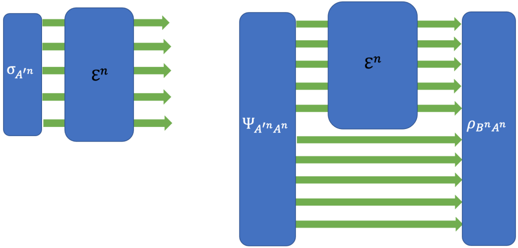


One-shot Quantum Capacity vs. Asymptotic Q. Capacity

	One-shot capacity	Asymptotic capacity
type of noise	arbitrary correlated	independent & identical
num. of uses n	finite	asymptotically infinite
error ϵ	nonzero error tolerance	vanishing

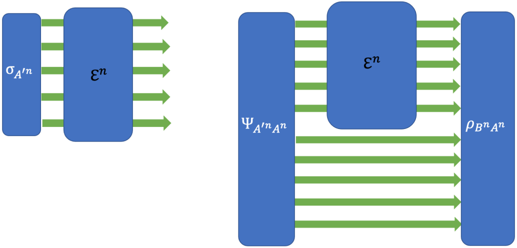
[1] Buscemi & Datta, IEEE TIT 56,1447 (2010)

Lower Bound on One-Shot Quantum Capacity



[2] Morgan & Winter, IEEE TIT 60,317 (2013); Tomamichel, Berta, & Renes, NC 7, 1 (2016); Furrer, Aberg, & Renner, CMP 306, 165 (2011)

Lower Bound on One-Shot Quantum Capacity



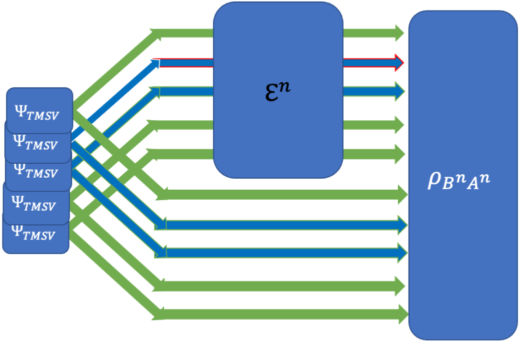
$$Q^{\epsilon}(\mathcal{E}^n) \geq \sup_{\eta \in \left(0, \sqrt{\epsilon/2}\right)} \left(-H_{\max}^{\sqrt{\epsilon/2}-\eta}(A^n|B^n)_\rho + 4\log_2 \eta - 2 \right),$$

Smooth max-entropy $H_{\max}^{\sqrt{\epsilon/2}-\eta}(A|B)_\rho$ is the minimum of $H_{\max}(A|B)_\rho$ over a neighbourhood of ρ with distance $\sqrt{\epsilon/2} - \eta$.

[2]Morgan & Winter, IEEE TIT 60,317 (2013); Tomamichel, Berta, & Renes, NC 7, 1 (2016); Furrer, Aberg, & Renner, CMP 306, 165 (2011)

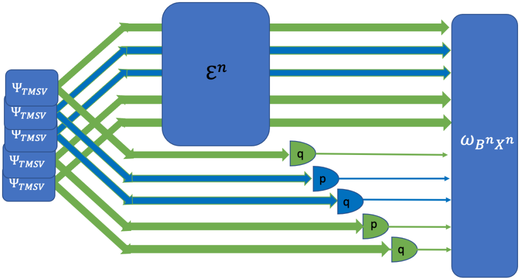
Lower Bound on One-Shot Quantum Capacity

k modes (blue arrows) are randomly chosen for test



$$Q^{\epsilon}(\mathcal{E}^{n-k}) \geq \sup_{\eta \in \left(0, \sqrt{\epsilon/2}\right)} \left(-H_{\max}^{\sqrt{\epsilon/2}-\eta}(A^{n-k}|B^{n-k})_{\rho} + 4\log_2 \eta - 2 \right),$$

Reduce to Estimate Max-Entropy of c-q State



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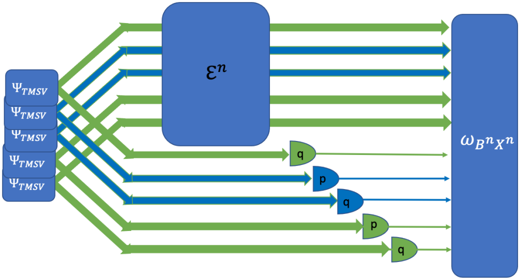
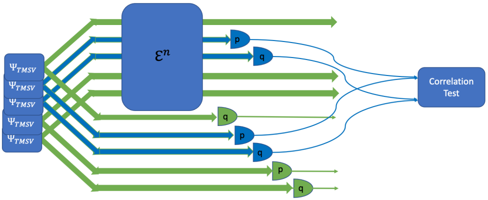


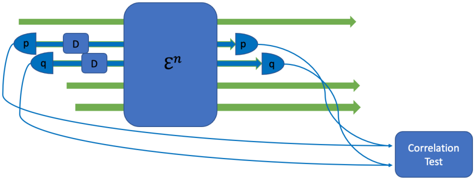
$$H_{\max}^{\sqrt{\epsilon/2}-\eta}(A^{n-k}|B^{n-k})_\rho \leq n\log_2\frac{d^2}{2\pi} + 2H_{\max}^{\zeta'}(X^{n-k}|B^{n-k})_\omega - 2\log_2\frac{2}{\zeta^2},$$

where $\zeta, \zeta' > 0$, $3\zeta + 5\zeta' = \sqrt{\epsilon/2} - \eta$, and d is the discretization width of continuous outcome.

Protocol for Arbitrary Correlated Noises



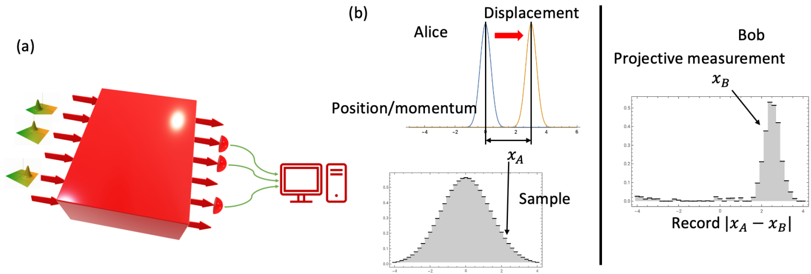
If the correlation test is passed, $H_{\max}^{\zeta'}(X^{n-k}|B^{n-k})_\omega$ is bounded by a function of average distance between outcomes



If the correlation test is passed, then with a small error probability (incorrect pass), we obtain a lower bound on Q^ϵ.

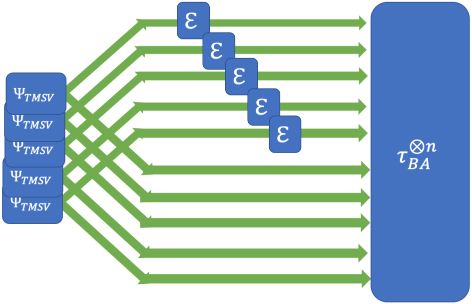
[3] Pfister, Rol, Mantri, Tomamichel, & Wehner, NC 9, 1 (2018).

Quantum Capacity Detection Protocol for General Noise



- Randomly select k uses for a test: if avg$|x_A - x_B| \leq t$, then pass the test
- Result of test is an inferred lower bound on # of qubits that can be transmitted with the remaining $n - k$ uses (with error probability $\leq p_{err}$)
- Smaller values of t results into higher values of capacity guaranteed by the test. However, low values of t make the test harder to pass.
- Transmission can be achieved by feeding half of a two-mode squeezed state into each of the $n - k$ uses

Identical & Independent Noise

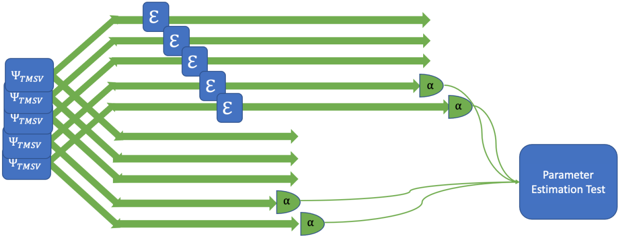


$$H^{\epsilon}_{\max}(A^n|B^n)_{\tau^{\otimes n}} \leq nH(A|B)_\tau + O\left(\sqrt{n\log(1/\epsilon)}\right).$$

$$Q^{\epsilon}(\mathcal{E}^{\otimes n}) \geq -nH(A|B)_\tau + \sup_{\eta\in\left(0,\sqrt{\epsilon/2}\right)}\left[4\log_2\eta - O\left(\sqrt{n\log\frac{1}{\sqrt{\epsilon/2}-\eta}}\right) - 2\right]$$

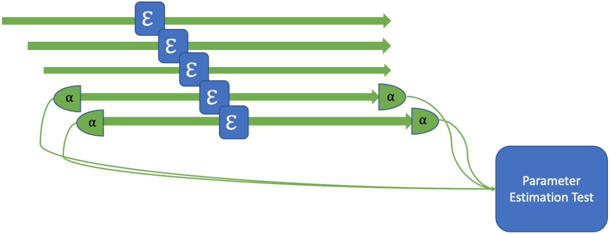
[4]Tomamichel, Colbeck, & Renner, IEEE TIT 55, 5840 (2009)

Protocol for Identical & Independent Noise



For i.i.d phase-insensitive Gaussian channels $\mathcal{E}^{\otimes n}$, that is, Gaussian channels $\mathcal{E}$ satisfying $\mathcal{E} \circ \mathcal{U}_\theta = \mathcal{U}_\theta \circ \mathcal{E}$ for every $\theta \in [0, 2\pi]$, where $\mathcal{U}_\theta$ is the unitary channel corresponding to $U_\theta = \exp[-i\theta\hat{n}]$, to estimate an upper bound of $H(A|B)_\tau$, only estimate $\langle \hat{q}_A^2 + \hat{p}_A^2 \rangle$, $\langle \hat{q}_B^2 + \hat{p}_B^2 \rangle$, and $\langle \hat{q}_A\hat{q}_B - \hat{p}_A\hat{p}_B \rangle$.

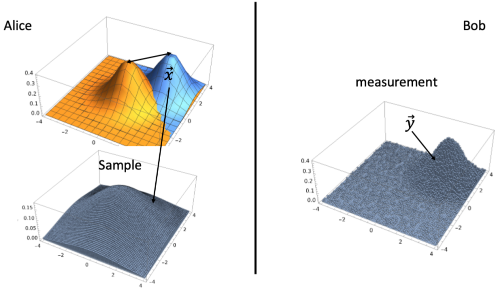
Protocol for Identical & Independent Noise



If the test is passed, with a small error probability (incorrect pass), we obtain a lower bound on Q^{ϵ}.

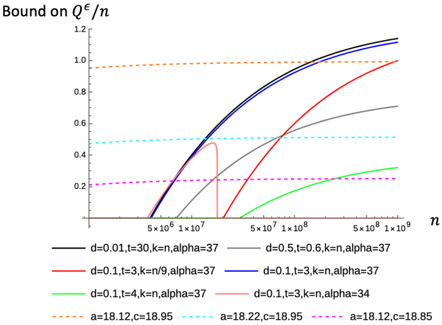
Quantum Capacity Detection Protocol for I.i.d Noise

This protocol provides better bound and does not require squeezing.



- test the correlations between $\boldsymbol{x}$ and $\boldsymbol{y}$, as well as the amount of noise added by the channel
- higher cross-correlations and lower added-noise witness higher values of the capacity

Numerical Simulation Results



Lower bound on capacity Q^ϵ/n vs number of modes n with $\epsilon = 10^{-3}$ and confidence level 99.9%

- lower bound given in the iid protocol converges much faster to the asymptotic limit than the general protocol.
- lower bound can be raised by increasing k/n, and/or by reducing d and/or by reducing t.
- cut-off parameter α should be chosen large enough

Summaries

- We have proposed two protocols for estimating lower bounds on quantum capacities of CV channels from experimental data
- The first protocol applies to arbitrarily correlated channels, while the second protocol is restricted to i.i.d. channels, has a lower sample complexity and requires simpler state preparations.
- Both protocols can be implemented using current technologies on optical platforms.
- They provide a flexible method to validate practical quantum communication devices and quantum memories, and to detect useful communication paths in dynamically changing quantum networks.