Restoring quantum communication efficiency over high loss optical fibres

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27/09/2022

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Based on :

- Mele F. A., Lami L., Giovannetti V., arXiv:2204.13128
- Mele F. A., Lami L., Giovannetti V., arXiv:2204.13129

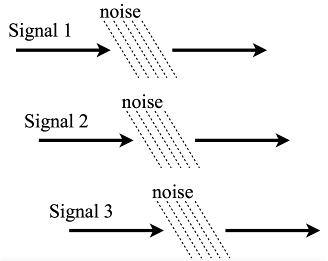
Goal:

Reliably transmitting qubits across long optical fibres.

Problem:

Until today, optical fibres have not been able to transmit qubits over long distances.

This impossibility comes from theoretical reasons which rest on the **memoryless assumption**.

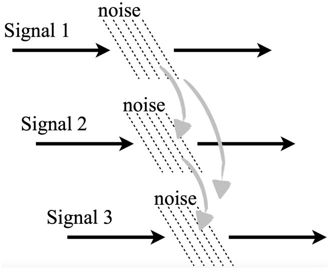


Possible known solution:

Exploiting quantum repeaters. However, this is expensive.

Our solution:

Exploiting memory effects.



Our main results

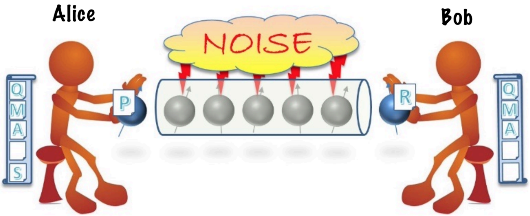
First main result:

By properly taking into account memory effects, unassisted quantum communication is possible at a fixed qubit rate for all non-zero values of the transmissivity of the fibre.

Second main result:

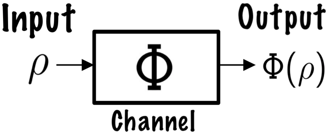
Entanglement assistance and the proper memory effects exploitation allows one to completely neutralise the effect of noise in classical communication.

Quantum Communication



The noise is mathematically characterized by a *quantum channel*.

$$\Phi : \mathfrak{S}(\mathcal{H}_{Alice}) \longmapsto \mathfrak{S}(\mathcal{H}_{Bob}) \ .$$

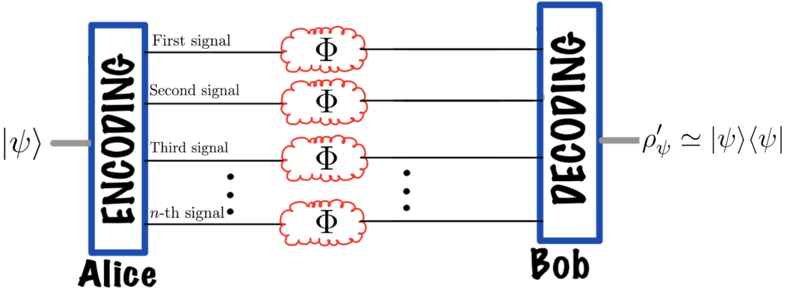


[3] Nielsen, Chuang, Cambridge University Press (2000)

Capacity of a quantum channel

Quantum information $\Longrightarrow$ $Q(\Phi) = \max_{\text{strategies with } P_{err} \to 0} \frac{\#\text{ qubit transferred}}{n}$

Classical information $\Longrightarrow$ $C(\Phi) = \max_{\text{strategies with } P_{err} \to 0} \frac{\#\text{ bit transferred}}{n}$



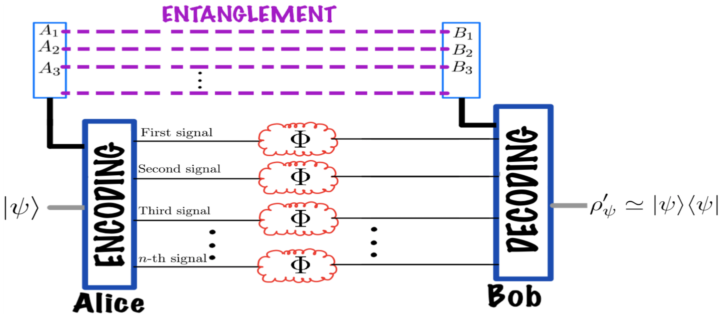
[4] Bennett, Shor, IEEE Trans. Inf. Theory (1998)
[5] Bennett et al., IEEE Trans. Inf. Theory (2002)

Extra resource: pre-shared entanglement

$C_{ea}(\Phi) \equiv$ entanglement-assisted classical capacity of Φ

$Q_{ea}(\Phi) \equiv$ entanglement-assisted quantum capacity of Φ

$$C_{ea}(\Phi) = 2Q_{ea}(\Phi) \quad (1)$$



Assumption: *memoryless noise model*

[6] Bennett et al., PRL (1999b)

General attenuator channels

Quantum carrier S: single-mode of electromagnetic radiation with a definite frequency and polarization.

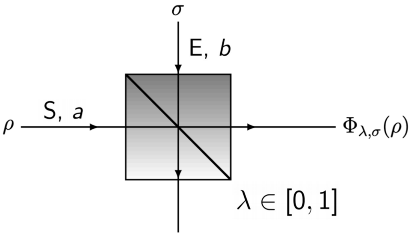
$$\mathcal{H}_E \equiv \mathcal{H}_S := L^2(\mathbb{R})$$

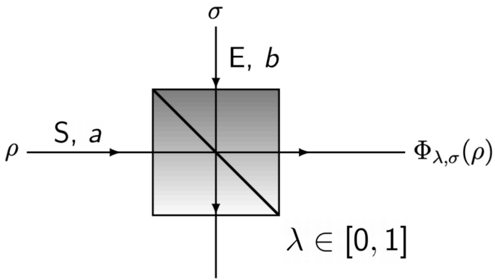
Fixed $\lambda \in [0,1]$ and $\sigma \in \mathfrak{S}(\mathcal{H}_E)$, a *general attenuator* $\Phi_{\lambda,\sigma}$ is defined by:

$$\Phi_{\lambda,\sigma}(\rho) := \mathrm{Tr}_E \left[U_\lambda^{(SE)} \; \rho \otimes \sigma \; U_\lambda^{(SE)\dagger} \right]$$

$$U_\lambda^{(SE)} := \exp\left[\arccos\sqrt{\lambda}\; (a^\dagger b - a b^\dagger)\right]$$

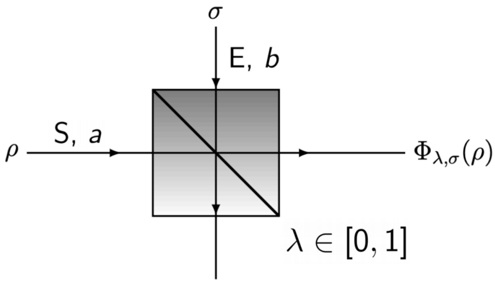
$$[a, a^\dagger] = \mathbb{1} \qquad [b, b^\dagger] = \mathbb{1}$$





- If $\lambda = 0$, then $\Phi_{0,\sigma}(\rho) = \sigma$, i.e. the channel is completely noisy;
- If $\lambda = 1$, then $\Phi_{1,\sigma} = \mathrm{Id}$, i.e. the channel is noiseless.

$\dim \mathcal{H}_S = \infty \Longrightarrow$ *Energy-constrained capacities:* $C_{\mathrm{ea}}(\Phi, N)$, $Q(\Phi, N)$



Memoryless optical fibres are represented by thermal attenuators.

$$\Phi_{\lambda,\tau_\nu} \equiv \text{thermal attenuator}$$

$$\tau_\nu := \frac{1}{\nu+1}\sum_{n=0}^{\infty}\left(\frac{\nu}{\nu+1}\right)^n |n\rangle\langle n| \qquad (2)$$

$$|n\rangle := \frac{1}{\sqrt{n!}}(b^\dagger)^n|0\rangle \equiv n\text{th Fock state} \qquad (3)$$

Already known results on the Quantum Capacity of $\Phi_{\lambda,\sigma}$

$$\boxed{Q(\Phi_{\lambda,\tau_\nu}) = 0 \quad \text{for all } \lambda \le 1/2}$$

Theorem (Die-Hard Quantum Communication)

For all $\lambda \in (0,1]$ there exists an environment state $\sigma(\lambda)$ such that

$$Q\left(\Phi_{\lambda,\sigma(\lambda)}\right) \ge Q\left(\Phi_{\lambda,\sigma(\lambda)}, 1/2\right) \ge c > 0 \ ,$$

where c is a constant.

Environment manipulation $\Rightarrow$ Quantum communication is possible even if $\lambda \to 0^+$

[7] Lami, Plenio, Giovannetti, Holevo, PRL (2020)

Our results

EA communication for vanishing transmissivity

Conjecture 1 (EA version of Die-Hard Quantum Communication)

For all $\lambda \in (0,1]$, $N > 0$, if $n \in \mathbb{N}$ is sufficiently large, then

$$C_{\mathrm{ea}}\left(\Phi_{\lambda,|n\rangle\langle n|}, N\right) \geq C(\mathrm{Id}, N)\ . \qquad (4)$$

This conjecture is proved in the seemingly worst case $\lambda \to 0^+$.

$\Longrightarrow$ *By consuming pre-shared entanglement and controlling the environment state, it is possible to communicate with the same or even better performance than the unassisted case of absence of noise.*

Novel method to easily deal with calculations involving thermal attenuator

Calculating the output of the thermal attenuator Φ_{λ,τ_ν} is often cumbersome.

Master equation trick

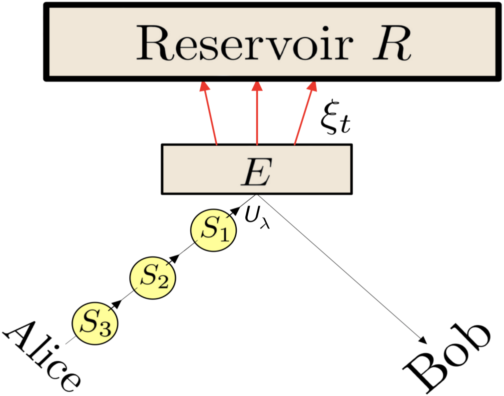
We derive a simple Kraus representation of Φ_{λ,τ_ν} exploiting the associated *Lindblad master equation*.

$\Rightarrow$ simple expressions of the output of Φ_{λ,τ_ν}.

(see Theorem 5 in [Mele F. A., Lami L., Giovannetti V., arXiv:2204.13129])

How to implement the control of the environment state?

Model of a non-memoryless optical fibre



The thermalisation map $\xi_{\delta t}$ satisfies for all $\sigma \in \mathfrak{S}(\mathcal{H}_E)$:

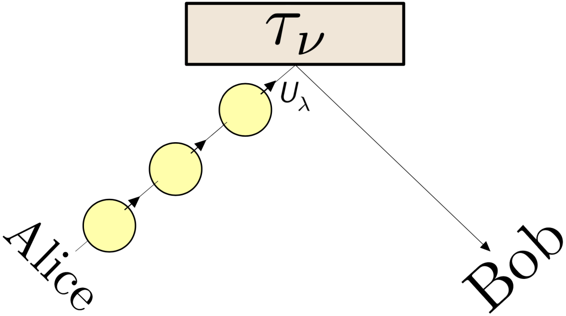
$$\xi_{\delta t}(\sigma) = \tau_\nu \quad \text{for } \delta t \geq t_E \,, \tag{5}$$

$$\xi_{\delta t}(\sigma) \simeq \sigma \quad \text{for } \delta t \ll t_E \,, \tag{6}$$

[8] Giovannetti, J. Phys. A (2005)

Memoryless assumption as a particular case

If the time interval δt between signals sent by Alice is such that $\delta t \geq t_E$, then the optical fibre is properly represented by a memoryless channel Φ_{λ,τ_ν}.

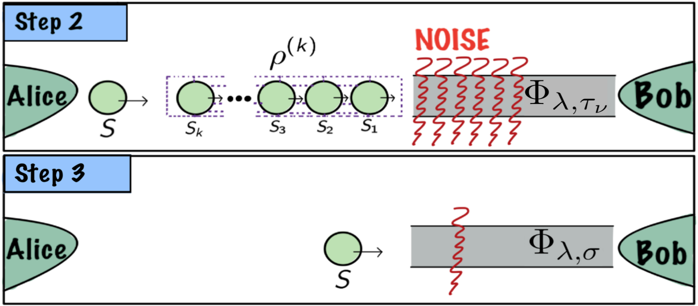


Noise attenuation protocol

Goal: Obtaining a memoryless channel $\Phi_{\lambda,\sigma}$ less noisy than Φ_{λ,τ_ν}.

Steps of the protocol:

- *Step 1*: Alice waits for a time t_E (so that the thermalisation resets E into τ_ν);
- *Step 2*: Alice sends k signals, dubbed 'trigger signals', separated by a time interval δt and initialised in a suitable k-modes state $\rho^{(k)}$.
- *Step 3*: After a time δt, Alice sends an information-carrying signal, which interacts with the environment altered in σ. Then, she goes back to step 1, unless the communication is complete.



We consider only those environment states σ which can be achieved by the noise attenuation protocol, i.e.

$$\sigma = \mathrm{Tr}_{S_1 \dots S_k}\left[\xi_{\delta t} \circ \mathcal{U}_\lambda^{(S_k E)} \circ \xi_{\delta t} \circ \mathcal{U}_\lambda^{(S_{k-1} E)} \dots \circ \xi_{\delta t} \circ \mathcal{U}_\lambda^{(S_1 E)}\left(\rho^{(k)} \otimes \tau_\nu\right)\right], \quad (7)$$

where $\mathcal{U}_\lambda^{(S_i E)}(\cdot) = U_\lambda^{(S_i E)}(\cdot)\left(U_\lambda^{(S_i E)}\right)^\dagger$.

Theorem (expensive version)

Let $\lambda \in (0, 1/2)$.
If Alice sends a sufficiently large number k of trigger signals separated by a sufficiently short δt and initialised in a suitable state, then the environment is altered in a state σ which satisfies

$$Q(\Phi_{\lambda,\sigma}) \geq Q(\Phi_{\lambda,\sigma}, 1/2) \geq \text{constant} > 0\,.$$

The maximum rate, achievable by adopting the noise attenuation protocol with k trigger signals sent to alter the environment into σ, is:

$$\frac{Q(\Phi_{\lambda,\sigma})}{k+1} \xrightarrow{k\gg 1} 0 \qquad (8)$$

Just two trigger signals is enough for λ small

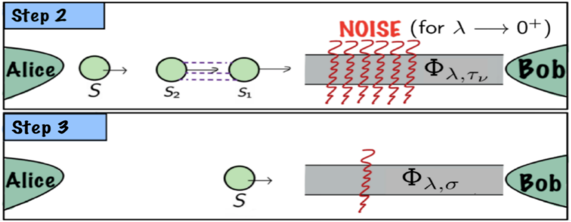
Theorem (cheaper version)

Let $\lambda > 0$ sufficiently small.
Suppose that Alice sends two trigger signals separated by a sufficiently short δt and initialised in

$$U^{(S_1 S_2)}_{\frac{1}{1+\lambda}} |0\rangle_{S_1} |n_\lambda\rangle_{S_2} \ , \tag{9}$$

where $n_\lambda \in \mathbb{N}$ is properly chosen. Then the environment is altered into a state σ such that

$$Q(\Phi_{\lambda,\sigma}) \geq Q(\Phi_{\lambda,\sigma}, 1/2) \geq \text{constant} > 0 \, . \tag{10}$$



The previous theorems can be generalised to the entanglement-assisted scenario

Let $\lambda \in (0, 1/2)$ and $N > 0$.
If Conjecture 1 is true, then the noise attenuation protocol allows one to alter the environment in a state σ such that

$$C_{\mathrm{ea}}\left(\Phi_{\lambda,\sigma}, N\right) \geq C(\mathrm{Id}, N) \ . \tag{11}$$

Conclusions

Memory effects can be engineered to improve the communication performance!

The noise attenuation protocol allows arbitrarily long optical fibres to reliably transmit:

- qubits at a fixed positive rate;
- bits and qubits at the maximum achievable rate in the noiseless scenario, provided that pre-shared entanglement is consumed.

Thank you for the attention!

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